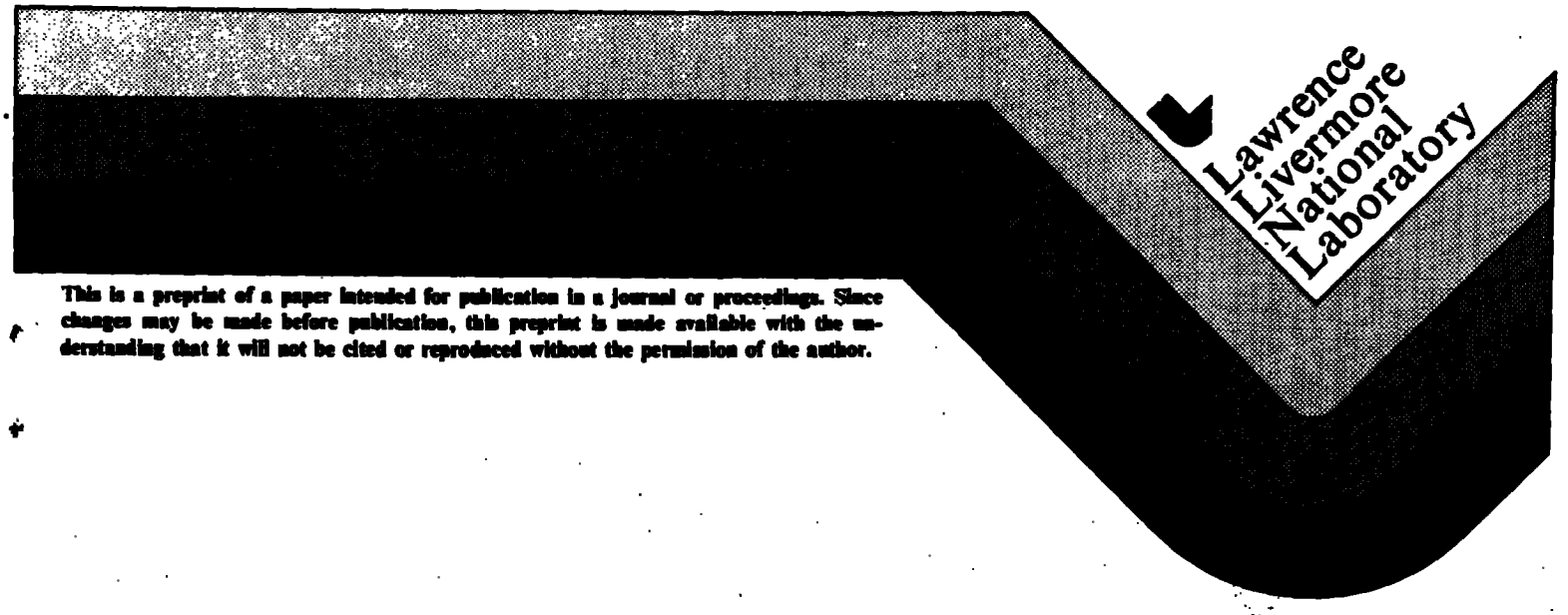
UCRL-94663
PREPRINT

LATTICE GAUGE THEORY AS A NUCLEAR MANY-BODY PROBLEM

G. J. Mathews, S. D. Bloom and N. J. Snyderman
University of California
Lawrence Livermore National Laboratory
Livermore, CA 94550

This paper was prepared for the
2nd Conference on the Intersections
between Particle and Nuclear Physics
May 26-31, 1986
Lake Louise, Canada

May 1986



DISCLAIMER

LATTICE GAUGE THEORY AS A NUCLEAR MANY-BODY PROBLEM

G. J. Mathews, S. D. Bloom, and N. J. Snyderman
University of California, Lawrence Livermore National Laboratory
Livermore CA, 94550

ABSTRACT

We discuss the conceptual connection between lattice quantum chromodynamics and a nuclear many-body problem. We begin with an illustrative example of how the O(3) nonlinear sigma model in (1+1) dimensions can be computed with a nuclear shell-model code with a speed which is competitive with other approaches. We then describe progress toward the implementation of this technology in lattice SU(2) Yang-Mills gauge theory.

INTRODUCTION

Since this is a conference on the intersections between particle and nuclear physics, we should begin with a discussion of how it is that lattice gauge theory occurs at such an intersection. From a particle theorist's point of view, lattice gauge theory, as proposed by Wilson,[1] is the only formulation of quantum chromodynamics which is amenable to numerical solution and thus, is a vehicle in which to analyze the theory of strong interactions. From a nuclear physicst's view point, such an ability to obtain numerical results in QCD will allow for a more fundamental understanding of nucleonic structure and interactions.

There is, however, another reason why this particle-physics problem is of interest to a nuclear physicist. That reason will be the main focus of this paper. It is that, at a computational level, the Hamiltonian formulation[2] of QCD lattice gauge theory is very similar to a problem which has been at the center of nuclear physics for many years; namely, the determination of the eigenstates of a nonrelativistic interacting many-body system of coupled angular momenta. In this paper, we develop this analogy and show, as an example, how our existing nuclear shell-model code can be applied, without modification, to a simple lattice theory, the O(3) nonlinear sigma model. This model has many properties in common with QCD. We also report on our progress toward adapting this approach to the more realistic SU(2) Yang-Mills pure gauge theory in (3+1) dimensions.

O(3) NONLINEAR SIGMA MODEL AND THE NUCLEAR SHELL MODEL

To illustrate the analogy between lattice QCD and a nuclear many-body problem, it is instructive to begin with a simpler lattice theory in which this equivalence is more transparent. The O(3) nonlinear sigma model in (1+1) dimensions has fundamental similarities to the SU(2) gauge theory in (3+1) dimensions. For example, it has a set of massless perturbative degrees of freedom (Goldstone bosons), the analogue of gluons in Yang-Mills gauge

theories, which do not appear in the spectrum of the theory. In both cases, only massive hadronic particles appear in the spectrum. The theory is also similar to SU(2) in that it is asymptotically free, has instantons, and is classically scale invariant with the consequent dimensional transmutation in the quantum field theory. Furthermore, the lattice O(3) model is computationally quite close to the SU(2) lattice gauge theory. As we shall see, both problems are excercises in the coupling of many angular momenta and the Hamiltonian matrix elements are, in both cases, linear combinations of 3n-j symbols.

The Lagrangian density for the nonlinear sigma model is,

$$L = (1/g^2)\partial_\mu \vec{\phi}\cdot\partial^\mu\vec{\phi} \quad , (\mu = 0,1) \tag{1}$$

where, the field variable, $\vec{\phi}$, is constrained by $\vec{\phi}\cdot\vec{\phi} = 1$. Thus, $\vec{\phi}$ is a three-component unit spin which can be parameterized by the spherical polar angles, θ and φ; $\vec{\phi} = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$. The discretized Hamiltonian[3] corresponding to equation (1) can be written,

$$Ha = (g^2/4)\sum_n \vec{L}_n^2 + (2/g^2)\sum_n (1 - \vec{\phi}_n\cdot\vec{\phi}_{n+1}) \tag{2}$$

where, a, is the lattice spacing and n labels the sites on a one-dimensional lattice. The quantum coordinates are θ and φ, and their conjugates give the kinetic-energy differential operator, $\vec{L}^2$ in Eq. (2), whose eigenfunctions are the spherical harmonics.

$$\vec{L}_n^2 Y_{\ell m}(\theta_n,\varphi_n) = \ell(\ell+1)Y_{\ell m}(\theta_n,\varphi_n) \tag{3}$$

The interaction term in equation (2) simply becomes,

$$\vec{\phi}_n\cdot\vec{\phi}_{n+1} = \frac{4\pi}{3}\sum(-1)^m Y_{\ell,m}(\theta_n,\varphi_n)Y_{\ell,-m}(\theta_{n+1},\varphi_{n+1}) \tag{4}$$

Therefore, by selecting a basis of angular momentum eigenstates one can immediately see an analogy with a nuclear shell-model problem, i.e. the Hamiltonian can be written,

$$H = \sum_i T_i + \sum_{i,j=i+1} V_{i,j} \tag{5}$$

where the "single-particle" orbitals correspond to sets of angular momentum eigenstates at each lattice site. The "single-particle" energies, T_i, now are $\ell(\ell+1)$, and the "two-body" potential corresponds to an interaction between adjacent lattice sites. This two-body potential is a simple dipole-dipole interaction, very similar to the quadrupole force often applied in shell-model calculations.

We have diagonalized the nonlinear sigma model Hamiltonian (Eq. 2) using the Livermore system of vectorized Lanczos-method shell-model codes[4]. Figure 1 compares our results for a 5 site lattice in a basis constructed by 3 operations of H on the

strong-coupling ground state (ℓ = 0 on all sites). This corresponds to a basis of up to 3707 m-scheme basis vectors. This figure shows the lattice mass gap (essentially the excitation energy of the L = 1 first excited state relative to the ground state in units of the lattice spacing) as a function of the coupling constant, $(2/g^2)$. Our results compare well with previous calculations based upon the Monte-Carlo functional integral[5] or a Hamiltonian variational calculation[6]. Even though we utilize a much smaller lattice we are able to obtain reasonable values for some distance into the weak-coupling regime. Using the Lanczos method in this way, seems to offer the advantage of fairly rapid convergence to the exact mass gap for a given basis. This calculation was run in about 100 sec of CDC 7600 cpu time per data point. Our good agreement may be somewhat fortuitous, however, since the results depend upon chosing the right combination of lattice and basis sizes. Nevertheless, these results have encouraged us to apply these same techniques to the more challanging Yang-Mills theories.

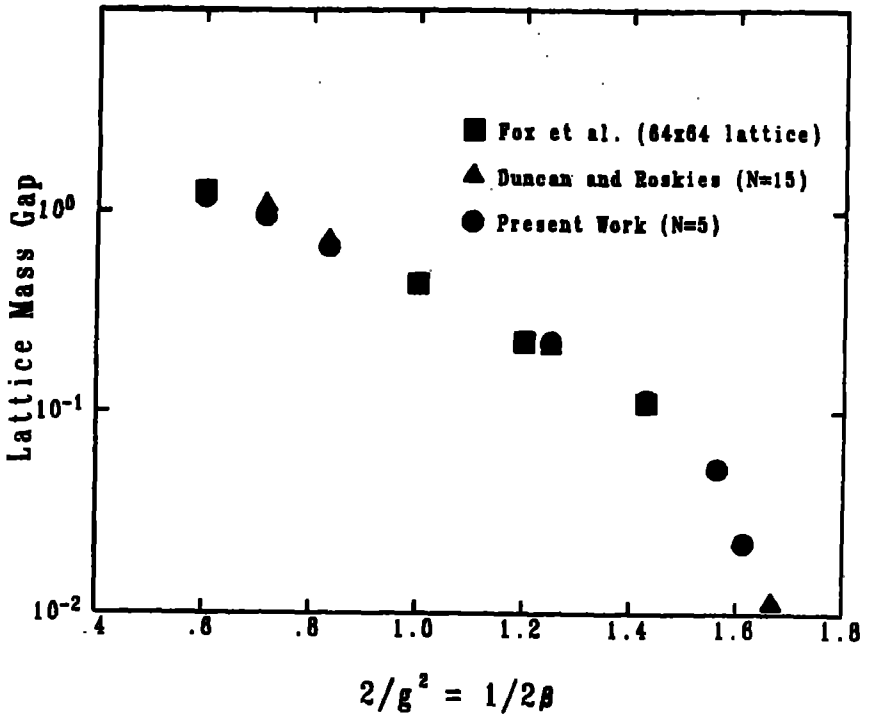


Fig. 1 Lattice mass gap for O(3) nonlinear sigma model in (1+1) dimensions. Results from our Lanczos-method shell-model code are compared with other techniques.

SU(2) LATTICE GAUGE THEORY IN (3+1) DIMENSIONS

Unlike QED without the electron field, which is a theory of free photons, the SU(2) gauge theory without quark fields is an interacting theory of closed-loop electric flux vorticies (glueballs). In the presence of quark fields, this electric flux

string can end on quark charges, making quark-antiquark mesons. (In the SU(3) theory, "Y" configurations of flux are also possible, which confine three quarks.)

In the lattice theory, on each link there is the field variable, $\exp(igAa)$, where the vector potential, $A = \vec{A}\cdot\vec{\tau}/2$, is a 2x2 matrix. The product of these operators around a closed loop on the lattice is the exponential of the magnetic flux through the loop. The vector potential becomes a set of angular variables, $\vec{\theta} = g\vec{A}a$, which are the quantum coordinates. In the gauge where the scalar potential is zero, the electric fields become canonical momenta and the Hamiltonian can be written,[2]

$$Ha = (g^2/2)\sum_{\ell}\vec{J}^2 + (2/g^2)(2 - \sum_{p} \mathrm{tr}U_1U_2U_3^{\dagger}U_4^{\dagger}) \qquad (6)$$

where, $\vec{J}^2$ is a kinetic-energy differential operator whose eigenfunctions are the Wigner D- matrices, and eigenvalues are $j(j+1)$, where $j = 0, 1/2, 1, 3/2, \cdots$. The interaction term, $\mathrm{tr}U_1U_2U_3^{\dagger}U_4^{\dagger}$, is a contraction of four D-matricies around a closed loop of four adjacent links on the lattice, a placquette. The basis states correspond to the excitation of angular momenta of links on the lattice for different numbers of placquettes under the addtional constraint (Gauss' law) that the the angular momenta of the links intersecting a given site must couple to zero.

Because of the large number of states associated with the implementation of a four-body operator, it is not feasible to treat this problem with an m-scheme code as we did with the nonlinear sigma model discussed in the previous section. With the constraint of Gauss' law, imposed by Clepsch-Gordan coefficients, the m's can be summed over, so the basis states are completely specified by the angular momenta on the links of the lattice. Therefore, the Hamiltonian SU(2) lattice gauge theory can also be thought of as a many-body system of coupled angular-momentum states. The main distinction from a nuclear many-body problem will be that the interaction term is a four-body operator.

Since the Lanczos algorithm[4] is based upon an iterative scheme involving successive operations of the Hamiltonian on a given start vector (the noninteracting strong-coupling ground state), we have found it convenient to rewrite the algorithm in terms of a nonlinear iterative combination of the diagonal matrix elements of the strong coupling ground state for powers of the Hamiltonian. These matrix elements can then be decomposed as a sum of products of the matrix elements of the Hamiltonian between arbitrary intermediate states. These intermediate matrix elements can be conveniently reduced (via angular-momentum coupling algebra) to the product of a Racah coefficient and two 9-j symbols around the four corners of a placquette. The problem then reduces to the calculation of these coupling coefficients.

A major simplification can be introduced by exploiting translational invariance on the lattice. The most efficient exploitation of this degeneracy has been proposed by Horn and Weinstein[7,8]. The problem of computation of $\langle H^n \rangle$ can be

reduced to the calculation of only terms which have a linear dependence on the volume, i.e. an overall translational degeneracy of N, where N is the number of placquettes on the lattice. These linear contributions correspond to connected graphs, (intersecting placquettes). The usual calculation of connected graphs involves excluded volume factors which are extremely difficult to compute. Horn and Weinstein[7,8] have found a way to circumvent this problem such that only manifestly connected graphs occur in the computation of the matrix elements. This approach can lead to an enormous simplification of the problem. For example, the calculation of $\langle H^4 \rangle$ on a 3x3x3 lattice requires 7695 intermediate states. This calculation can be reduced, however, to the calculation of 3 manifestly connected graphs. We are currently in the process of implementing this algorithm for the calculation of high powers of $\langle H^n \rangle$. We estimate that with these simplifications we should be able to calculate to order $\langle H^{20} \rangle$.

At present we have available matrix elements of up to $\langle H^{10} \rangle$ (basis of H^5) computed by Duncan and Roskies[6] using a different technique. This limited basis, however, only allows for 5 Lanczos iterations on the strong-coupling ground state start vector. Although with each iteration we are approaching closer to the weak coupling regime, many more Lanczos iterations, within this fixed basis, will be required before we approach the weak coupling regime. With a basis of up to H^{10}, and applying enough Lanczos iterations in this basis, we expect to reach the scaling regime necessary to obtain reliable calculations of the hadronic mass spectrum.

ACKNOWLEDGMENT

Work performed under the auspices of the U. S. Department of Energy by the Lawrence Livermore National Laboratory under contract number W-7405-ENG-48.

REFERENCES

1. K. G. Wilson, Phys. Rev. D10, 2435 (1974).
2. J. Kogut and L. Susskind, Phys. Rev., D11, 395 (1975).
3. C. J. Hammer, J. B. Kogut, and L. Susskind, Phys. Rev., D19, 3091 (1979).
4. G. J. Mathews, S. D. Bloom, K. Takahashi, G. M. Fuller, and R. F. Hausman, in "Nuclear Shell Models", M. Vallieres and B. H. Wildenthal, eds., (World Scientific; Singapore) (1985) pp. 447-457.
5. G. Fox, R. Gupta, O. Martin, and S. Otto, Nucl. Phys., B205, 188 (1982).
6. A. Duncan and R. Roskies, Phys. Rev., D31, 364, (1985).
7. D. Horn and M. Weinstein, Phys. Rev., D30, 1256 (1984).
8. D. Horn, M. Karliner, and M. Weinstein, Phys. Rev., D31, 2589 (1985).